

MODELING A STIFF-INPLANE TILTROTOR USING TWO MULTIBODY ANALYSES: A VALIDATION STUDY

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Abstract

The objective of this investigation is to illustrate a modular approach in the development and validation of sophisticated rotorcraft analytical models within the framework of multibody dynamics simulations. This approach is demonstrated with the development of a stiff-inplane tiltrotor wind-tunnel model using two multibody dynamics analyses. The two multibody dynamics codes used are: DYMORE and MBDyn, which are both capable of supporting comprehensive, multibody-based rotorcraft modeling and dynamic simulation. The multibody dynamics models developed in this effort include the gimballed hub, rotor blades, pitch links, swashplate, conversion actuators which are attached to the pylon, and the elastic wing. The natural frequencies, mode shapes, and stability characteristics of key sub-component structures and the kinematic couplings of rotor system are correlated with the predictions of the two analyses. Furthermore, experimental data obtained in ground vibration tests and wind-tunnel tests is extensively used to validate the analytical models. The validated models are then used to predict the tiltrotor whirl-flutter stability boundary, which shows good agreement with the experimental measurement.

Introduction

Rotorcraft, as exemplified in a tiltrotor, consist of complex mechanical and structural systems that include interconnected rigid and deformable components. The dynamics of such large-scale, multibody systems, which are further loaded with unsteady aerodynamic forces, is highly nonlinear, non-periodic, and difficult to predict (Ref. 1). Multibody Dynamics Simulation (MDS) methodology, which is now widely used in automobile, railway vehicles, robotics, and fixed-wing aircraft industries, has been adopted for rotorcraft applications in recent years (Ref. 2–6). Coupled with Computational Fluid Dynamics (CFD) analyses, which provide accurate predictions of the aerodynamic forces, Computational Structural Dynamics (CSD) models of rotorcraft based on MDS establish a high fidelity simulation framework for studying rotorcraft aeromechanics (Ref. 7).

Developing rotorcraft models using multibody dynamics analyses presents unique features when compared with using classical comprehensive rotorcraft analyses. Multibody dynamics analyses provide elementary structural components such as rigid body, beam, cable, and shell elements as well as constraint components including the revolte hinge, prismatic joint, spherical joint, and boundary condition constraint. These features facilitate the development of sophisticated structural rotorcraft models component by component, as well as the validation of the model component by component. Modularization in model development and validation is a reliable method to build the model accurately and efficiently.

This paper presents a validation study of multibody analytical models of a stiff-inplane tiltrotor. The present model development and validation procedure are based on this modularization concept which is enabled by MDS. Two multibody dynamics codes are used: DYMORE (Ref. 8) and MBDyn (Ref. 9), which are both capable of supporting comprehensive, multibody-based rotorcraft modeling and dynamic simulation.

Analytical Models

The model-scale tiltrotor studied in this paper, known as the Wing and Rotor Aeroelastic Testing System (WRATS), has a stiff-inplane, three-bladed rotor system and was tested at the NASA Langley Transonic Dynamics Tunnel (TDT) in 2000. Ref 10 presents in detail the ground vibration test and wind tunnel test of the stiff-inplane tiltrotor semi-span model, from which results are used in this validation study. The main objective of both the experimental wind-tunnel test and the MDS model development discussed in this paper is to determine the whirl-flutter stability boundary of the WRATS tiltrotor configuration. The ultimate goal of MDS is to calculate the tiltrotor whirl-flutter speed accurately and consistently when compared to experimental results. Thus, the analytical simulations may eventually supplant the expensive and time consuming experimental verifications. The accurate prediction of whirl-flutter speed is, however, a challenging task because it is an “integral” result which depends on the precise modeling of all the important sub-component structures, such as wing, pylon, and rotor system. A systematic approach is to encapsulate these sub-component structures into “modules” which can be validated separately and then assembled into the final model.

Two Multibody Dynamics Codes

Analytical models have been developed using two MDS analyses known as DYMORE and MBDyn. The two codes have similar capabilities, but in order to test the robustness of the MDS methodology and the feasibility of the modular approach in modeling the stiff-inplane tiltrotor, their results were compared with models created by different researchers at different institutions.

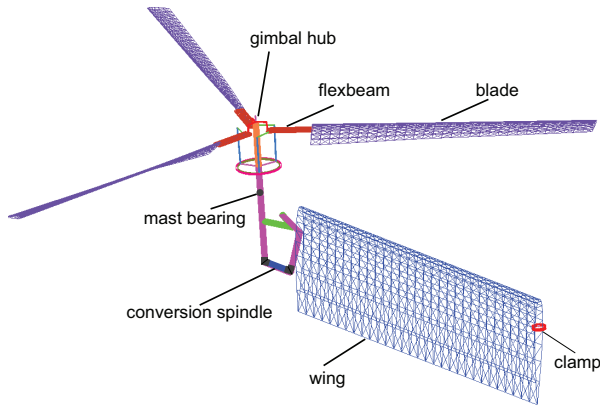
DYMORE is a finite element based tool for the analysis of nonlinear flexible multibody systems, developed at the School of Aerospace Engineering, Georgia Institute of Technology. Its element library includes rigid and deformable bodies as well as joint elements. Deformable bodies are modeled with the finite element method, and the formulations of beams and shells are geometrically exact, i.e. they account for arbitrarily large displacements and finite rotations, but are limited to small strains. A modal super-element is provided to construct complex elastic structure. It is assumed that the unconstrained mass and stiffness matrices of an elastic structure have been obtained from a finite element package such as NASTRAN. Herting’s transformation will be applied to those matrices to obtain a set of reduced mass and stiffness matrices that can be input to DYMORE. The equations of equilibrium are derived in a Cartesian inertial reference, and constraints are modeled using the Lagrange multiplier technique. This leads to a system of Differential Algebraic

Equations (DAE) which is then solved using a robust time integration scheme (Ref. 11). The aerodynamic forces can be computed with the built-in lifting line theory or through the coupling with an external CFD code.

MBDyn is a general-purpose MDS software, developed at the Dipartimento di Ingegneria Aerospaziale of the University “Politecnico di Milano”, Italy. It features the integrated multidisciplinary analysis of multibody, multiphysics systems, including nonlinear mechanics of rigid and flexible constrained bodies, smart materials, electric networks, active control, hydraulic networks, essential fixed-wing and rotorcraft aerodynamics. It allows to simulate the behavior of heterogeneous mechanical, aeroservoelastic systems based on first principles equations. The deformable components in MBDyn include lumped components, geometrically exact, nonlinear beams which utilize a C^0 beam discretization based on the finite volume concept. A general-purpose component mode synthesis element is also provided in MBDyn. The DAE are solved using an original implicit multi-step integration scheme that allows to tune the algorithmic dissipation, and achieves second-order accuracy (Ref. 12). MBDyn provides several aerodynamic models with different levels of fidelity, which include the blade element theory and coupling with an external CFD code.

WRATS Stiff-Inplane Tiltrotor

The multibody dynamics models developed in this effort include the gimballed hub, rotor blades, pitch links, swashplate, conversion actuators which are attached to the pylon, and the elastic wing (Fig. 1.) Both DYMORE and MBDyn models of the stiff-inplane rotor also include a model of the constant-velocity (CV) joint in the hub. This CV joint allows the rotating shaft to transmit power to the rotor through a gimballed angle at nearly constant rotational speed. The aerodynamic forces on the rotor and wing are modeled with aerodynamic elements based on lifting line theory in the DYMORE model. The wing has five aerodynamic elements while each blade uses four aerodynamic elements. The MBDyn model of rotor aerodynamics uses the built-in blade element theory. The rotor inflow model is not included in this study because the tiltrotor induced inflow is negligible in the high speed airplane-mode flight and with the rotor windmilling (These are the test conditions where tiltrotor whirl flutter is critical, and hence, these conditions are the focus of the current analytical simulations.) The aerodynamic interactions between rotor and wing are also neglected in both analytical models.



(a) WRATS stiff-inplane tiltrotor (DYMORE model shown.)

Property	Value
Rotor Type	Gimballed, stiff-inplane
Number of Blades	3
Rotor Diameter	7.6 ft
Twist	47.5°
Hover RPM	888
Cruise RPM	742
Lift curve slope (nom.)	5.9
Solidity	0.105
Geometric δ_3	-15.0°
Wing Span	55.0 in
Wing Chord	20.0 in
Wing Sweep	-6.0°
Wing Dihedral	3.5°

(b) Key parameters of the WRATS stiff-inplane tiltrotor (Ref. 10).

Figure 1: WRATS stiff-inplane tiltrotor model.

Modular Model Development Approach

This section describes the modular approach which is facilitated by MDS, and enables a systematic validation process of the model. This process allows for the development of MDS models component by component and the correlation of the analytical models at each step. Fig. 2 illustrates the modular procedure applied to the semi-span WRATS stiff-inplane tiltrotor model. The model can be naturally divided into two sub-systems, namely, the fixed sub-system and rotating sub-system which are developed and validated separately.

The development of the *rotating system* starts with a *clamped blade* model which consists only a clamped beam (Fig. 3a.) This *clamped blade* model is then connected to the gimbal hub through a flexbeam, torque tube, and control system (pitch link, pitch horn, and swashplate) to construct the *single blade* model (Fig. 4a.) This *single blade* system is then extended to include multiple blades to complete the *isolated rotor* system (Fig. 6a.) The capa-

bility of replicating blades is available in both DYMORE and MBDyn analyses.

The development of the *fixed system* starts with a *clamped wing* model which consists only a clamped beam, similar to the clamped blade model. This *clamped wing* model is then enlarged to include the rotor pylon system consisting of conversion actuators and downstop lock devices to form the *wing/pylon* model. Further, a dummy rigid body, with mass equal to that of the rotor system, is added to complete the *wing/pylon [rotor weight]* model (Fig. 8a.)

The WRATS stiff-inplane tiltrotor *semi-span structural model* is completed by joining the fixed and rotating sub-systems. Further, aerodynamic elements are added to the two sub-systems (wing aerodynamics in the fixed sub-system, and blade aerodynamics in the rotating sub-system.) After validation at the sub-system level, the aerodynamic elements are merged to the semi-span structural model to complete the full WRATS stiff-inplane *tiltrotor model*.

Validation Procedure and Results

This section presents the simulation and validation of individual key sub-components of the semi-span stiff-inplane tiltrotor and the final integral results of the complete model.

Validation of Rotating System Model

The results used to validate the rotor sub-system are: (i) natural frequencies for validation of the clamped blade, (ii) natural frequencies and elastic modes for validation of the single blade (with flexbeam and control system added), (iii) natural frequencies and pitch-flap coupling (δ_3) for validation of the isolated rotor system.

The first step in the overall model development is to determine the crucial elastic structures in the system, and then to decide what type of elastic elements should be used to model them. For example, in the WRATS stiff-inplane tiltrotor system, the main elastic structures are the wing bar and blade (including main blade and flexbeam), and they can all be modeled with beam element in the MDS analyses. Once the elastic elements are selected, they may be constructed with a routine procedure in the MDS analyses. Taking the blade model as an example, we set it up with a single elastic beam clamped at the root (Fig. 3a.) When building the beam model, it is important to verify that the sectional inertial, stiffness, geometrical (i.e. twist angle in the blade case) properties are properly compiled. A convergence study may be carried out by refining the beam discretization to determine the number of elements required to achieve desired accuracy. Natural

The Modular

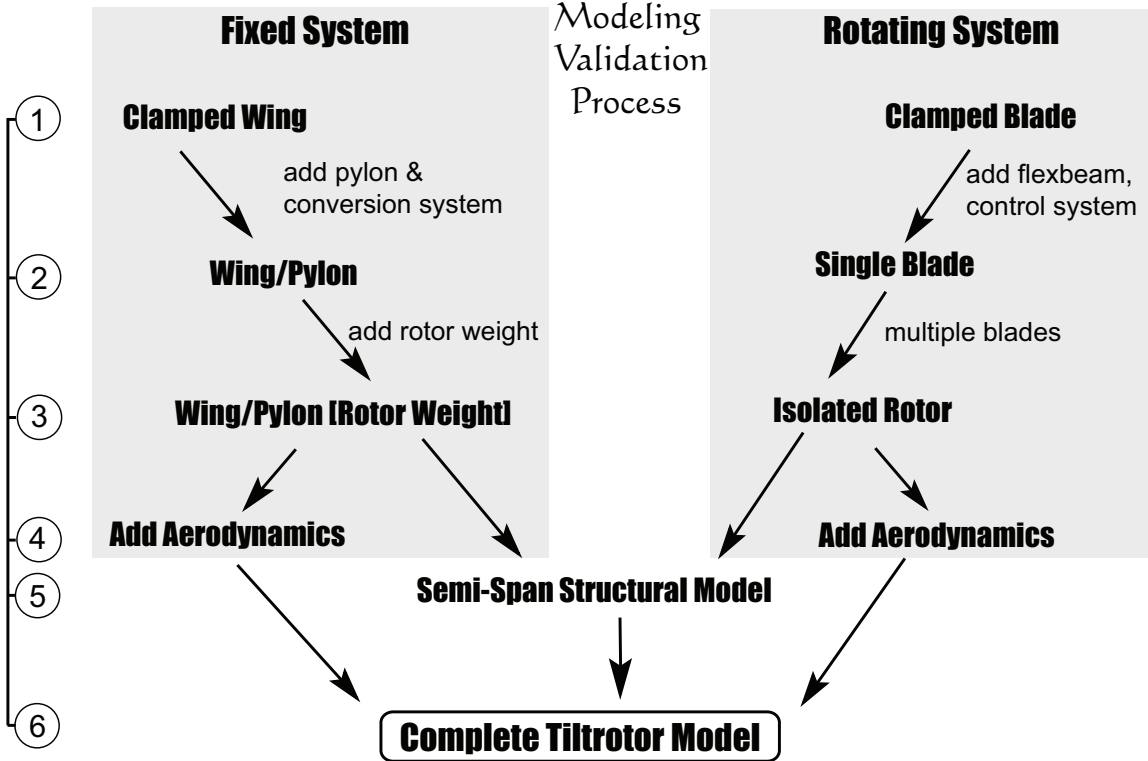


Figure 2: The modular modeling and validation process of WRATS stiff-inplane tiltrotor.

frequencies and mode shapes may be used as indicators of the model accuracy. Table 3b presents the predictions of the natural nonrotating frequencies of the clamped main blade in vacuum. The analytical results show good agreement with the test results, which verified the MDS models of the main blade beam.



(a) Clamped blade model (MBDyn model shown.)

Mode	Exp	MBDyn	DYMORE
1st Flap	12.29	12.29	12.08
1st Inplane	34.11	34.65	34.30
2nd Flap	52.44	56.08	56.25
1st Torsion	113.35	114.30	115.18

(b) Clamped blade frequencies, Hz.

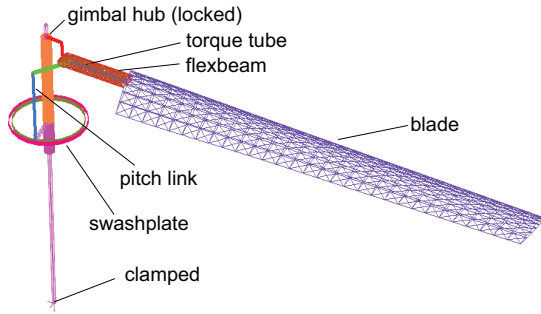
Figure 3: Clamped main blade model.

The clamped main single blade model is then expanded by connecting the main blade to the gimbal hub through a flexbeam, and connecting to the control system through torque tube, and pitch links (Fig. 4a.) The new flexible elements are the flexbeam and a linear spring along the pitch

link to simulate the control system stiffness. The stiff-inplane tiltrotor is designed so that the flexbeam transfers the flapwise and chordwise bending moments and the three forces (axial, lateral, and vertical) to the hub, while the torque tube carries the torsional moments (in addition, small part of the vertical force is transferred through the pitch links.) Various mechanical joints, i.e. prismatic and spherical joints, are used to achieve the load multi-path design. These mechanical joints are readily modeled with the corresponding joint elements in the MDS analyses, which modify the degrees of freedom of the connection nodes.

The validation results of this single blade model are shown in Figs. 4 and 5. Table 4b shows the natural nonrotating frequencies of the single blade in vacuum with the blade collective at 75% radius set to 10 deg (gimbal locked.) Good agreements are seen between the MDS predictions and the test data in general. However, compared with the test data, the MDS predictions of the 1st inplane mode frequency presents an 8% error, though the DYMORE and MBDyn predictions are close to each other. This may indicate the blade properties used in the MDS analyses present some small discrepancies compared to the actual properties of the blade used in the experiment. Figure 5 shows the predictions of the first four natural

mode shapes of the single blade system, whose frequencies are listed in Tab. 4b. Good agreements are seen between DYMORE and MBDyn predictions.



(a) Single blade system (DYMORE model shown.)

Mode	Exp	MBDyn	DYMORE
1st Flap	6.6	6.87	6.83
1st Inplane	19.3	18.90	18.83
2nd Flap	26.6	26.42	26.78
3rd Flap	68.8	70.35	71.78
1st Torsion	114.5	113.00	110.57

(b) Frequencies of single blade system, Hz.

Figure 4: Single blade model (gimbal locked.)

The single blade model is then enlarged to include all three blades (Fig. 6a.) The validation of this isolated full rotor model is accomplished, again, by verifying the predicted natural frequencies with test data (Tab. 6b.) Furthermore, Figure 6c compares the DYMORE and MBDyn predictions of the kinematic pitch-flap coupling (δ_3), and shows good agreement. Precise geometric input, i.e the positions of the pitch link and pitch horn connection nodes, ensures correct kinematic couplings in the rotor system.

The aerodynamic elements are then added to the completed rotor structure model. Figure 7 compares the predictions of the blade collective pitch of the rotor windmilling with the test data. Windmilling condition is obtained in wind tunnel test by disconnecting the rotor from the drive system, and adjusting the rotor collective pitch angle to reach a prescribed rotor rotating speed. As the rotor is unpowered, the rotor mast aerodynamic torque is approximately zero under windmilling condition. Results corresponding to two rotor speeds are shown: 888 RPM (hover rotor speed), and 742 RPM (maximum performance cruise flight rotor speed.) The MDS predictions of the windmilling rotor collective angle compare well with test data for both rotor speeds.

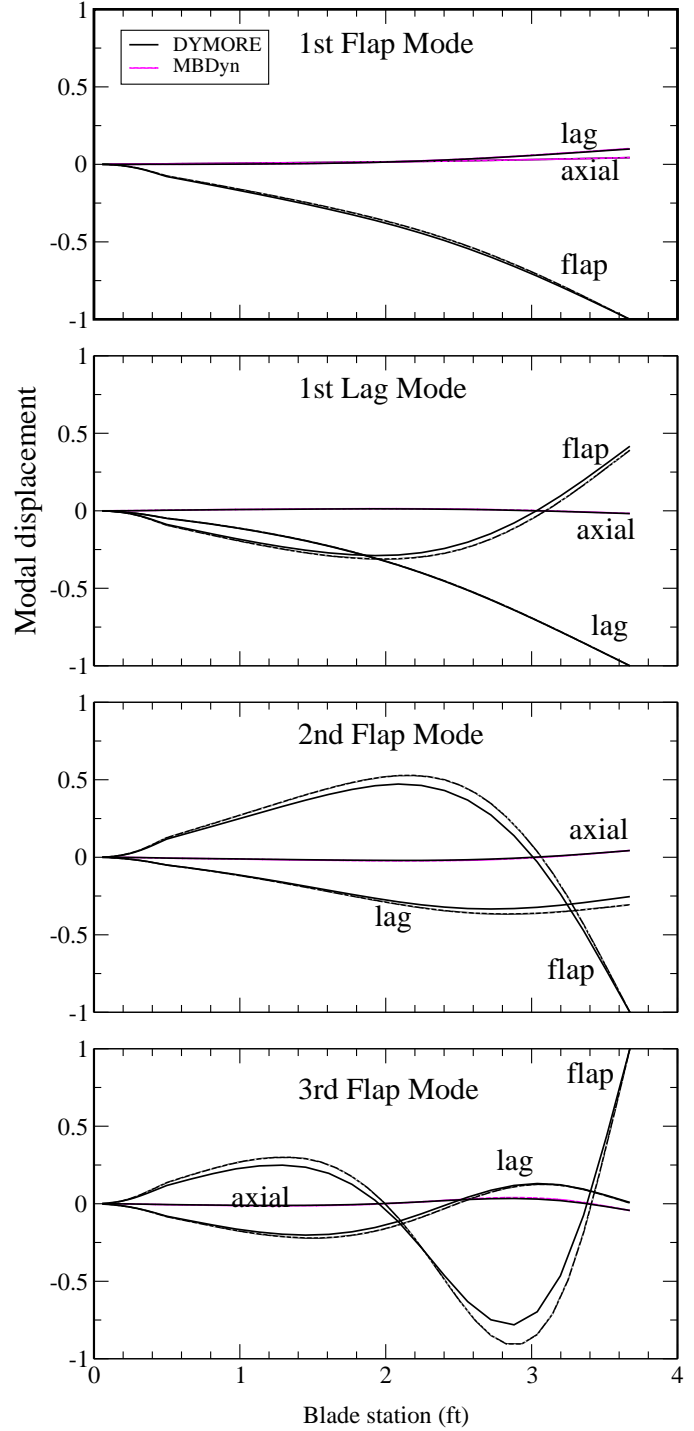
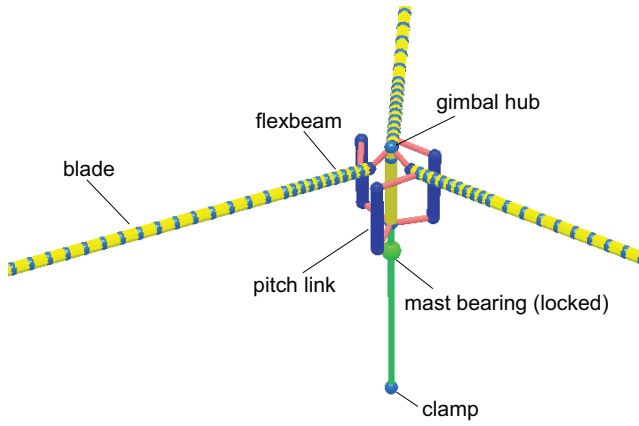


Figure 5: Natural mode shapes of the single blade system.

Validation of Fixed System Model

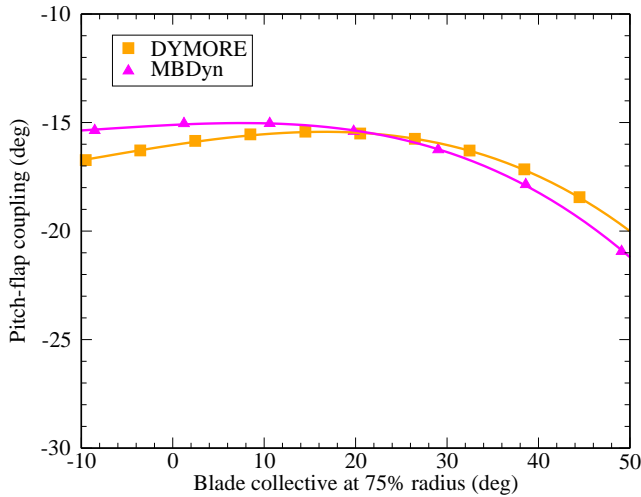
The results used to validate components of the non-rotating system are: (i) natural frequencies and elastic modes for validation of the clamped wing/pylon system, (ii) aeroelastic stability results for validation of the same



(a) Isolated rotor (MBDyn model shown.)

Mode	Exp	MBDyn	DYMORE
Gimbal	2.0	2.31	2.33
Cone	6.8	6.62	6.80
1st Lag	19.7	18.38	18.74
2nd Flap	25.0	26.11	26.69
1st Torsion	112.1	112.75	110.57

(b) Full rotor frequencies (non-rotating, free gimbal), Hz.



(c) Pitch-flap coupling (δ_3 .)

Figure 6: Isolated rotor model.

wing/pylon system with wing aerodynamics included.

The modeling of the fixed sub-system begins with a clamped wing beam model which is validated similarly to the clamped blade model. This wing model is then extended to include the pylon system and a dummy rotor. Fig. 8a illustrates the components in the fixed system model. The module contains all the components in the fixed system, such as the wing, pylon, and conversion actuators. The entire rotating system is replaced by a rigid body with equal weight. The key validation pa-

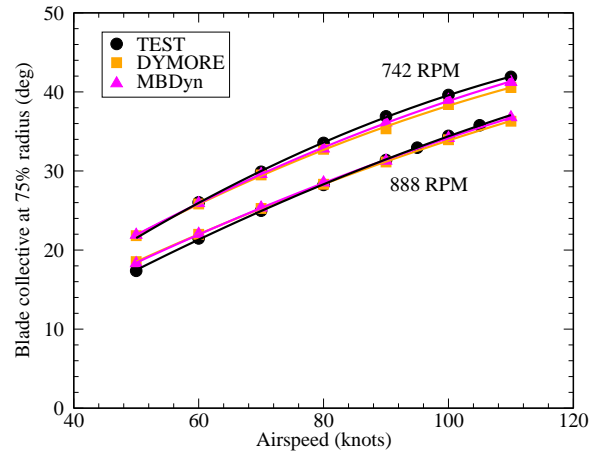
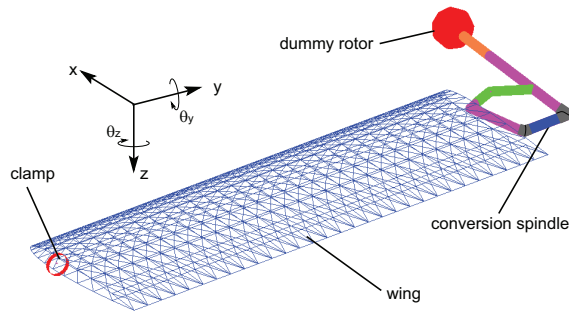


Figure 7: Variation of blade collective pitch of rotor wind-milling with airspeed.

rameters are the predictions of natural frequencies of the wing/pylon sub-system and the mode shapes at the rotor hub. These frequencies and mode shapes are known to have a major influence on the overall aeroelastic stability characteristics of the complete tiltrotor model.

Fig. 8 presents the validation of the nonrotating system model. It shows relatively good agreement between the MDS predictions of the frequencies and mode shapes with the experimental data, except a slight underprediction in the wing beam frequency. This difference is caused by small discrepancies in the flexibility properties of conversion actuators used in the MDS models. The pylon/conversion system, as is common with a tiltrotor, is very complex. There are two possible pylon configurations when the tiltrotor is converted to airplane-mode. One is the off-downstop configuration, which simulates the pylon conversion actuator stiffness and damping in helicopter mode and during conversion. The other is the on-downstop configuration, which has higher pylon conversion actuator stiffnesses to simulate the “locked” pylon state after conversion to airplane-mode. The flexibilities of the conversion actuators and pylon downstop locking device must be carefully captured in the MDS analyses using a flexible joint element, which consists of a set of collocated concentrated springs and dampers. Together with the precise input of the inertia and stiffness properties of the wing beam, it ensures a correct modal representation of the fixed sub-system. Nonetheless, the predicted natural frequencies and mode shapes at the rotor hub show satisfactory agreements with the measurements from the Ground Vibration Test (GVT) (Ref. 10). Thus, the fixed system model is considered validated and will be assembled with the rotating sub-system into the final tiltrotor structural model.

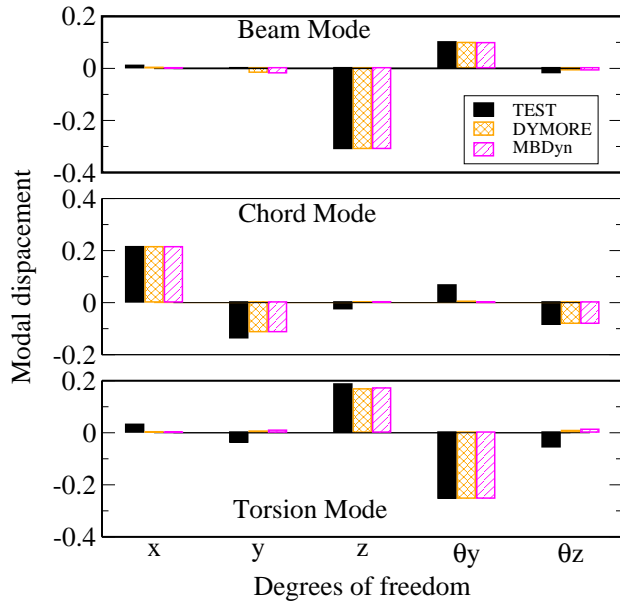
The wing aerodynamics model is verified by comparing



(a) Wing/pylon model (DYMORE model shown.)

Mode	Exp	MBDyn	DYMORE
Beam	5.43	5.32	5.30
Chord	8.11	8.31	8.32
Torsion	10.54	10.45	10.57

(b) Frequencies of wing/pylon system (off-downstop.)



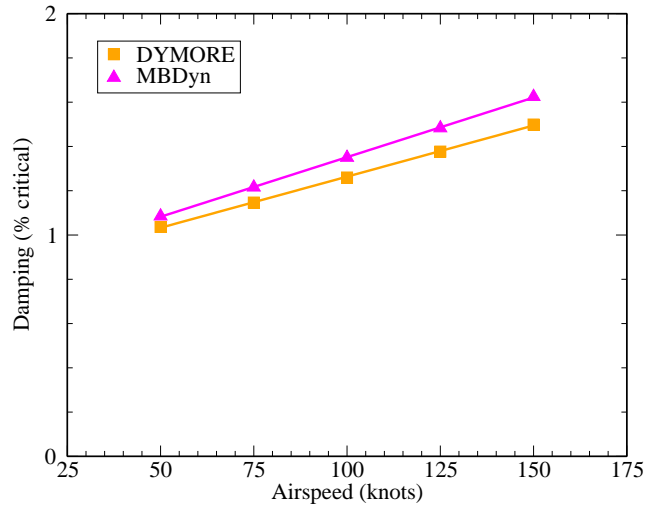
(c) Mode shapes at rotor hub.

Figure 8: Validation of wing/pylon sub-system model (off-downstop configuration.)

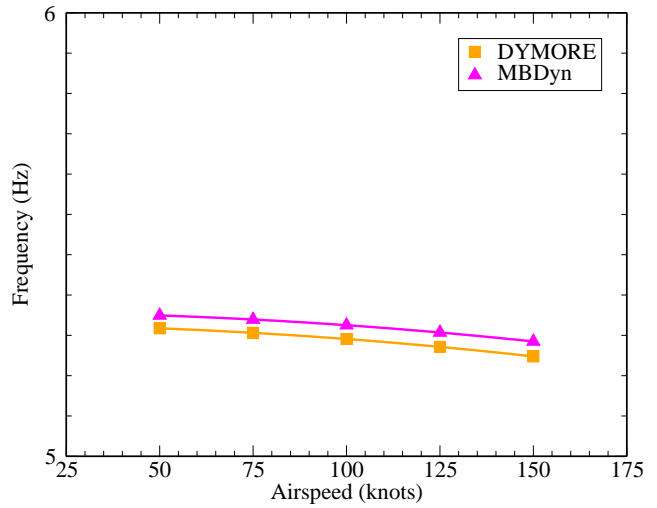
the aeroelastic stability characteristics of the wing/pylon [rotor weight] sub-system between DYMORE and MBDyn predictions. Figure 9 illustrates the predicted variation of the wing beam-wise mode damping and frequency with airspeed, and shows good agreement between DYMORE and MBDyn calculations.

Validation of Complete Semi-Span Model

The integral numerical simulations of the complete semi-span tiltrotor model is the whirl-flutter stability boundary



(a) Wing beam mode damping versus airspeed.



(b) Wing beam mode frequency versus airspeed.

Figure 9: Wing beam frequency/damping variation with airspeed (wing/pylon/dummy-rotor system.)

determination in airplane mode (Fig. 10.) Fig. 10a shows the WRATS stiff-inplane tiltrotor in TDT tunnel. Fig. 10b illustrates the complete semi-span tiltrotor MDS model. Comparing with the wind tunnel model, the MDS model is shown to contain all the critical components which are constructed and verified using the modular approach.

Figs. 10c to 10f show the variation of wing beam mode frequency and damping characteristics with tunnel airspeed for the off-downstop pylon stiffness configuration at steady rotor speeds of 742 and 888 RPM. The DYMORE and MBDyn predictions for sub-critical damping and flutter speed compare relatively well for both rotor speeds, but slightly under-predict the wing frequency in both the 742 and 888 RPM cases. This underprediction has been previously noted in the validation of the fixed system model.

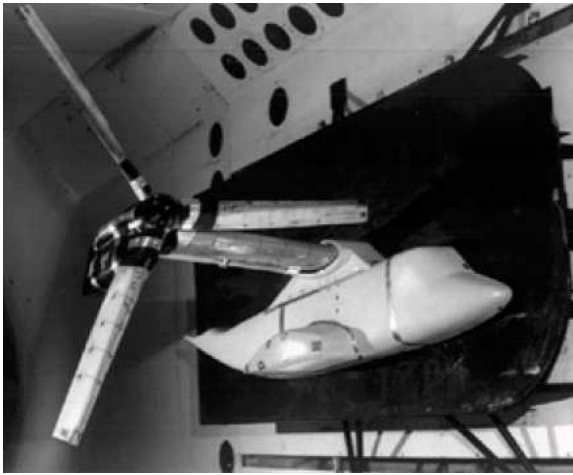
The prediction accuracy may be further improved by using more refined structural properties, for example by refining the flexibility of conversion actuators. Additional results on whirl flutter stability of the WRATS stiff-inplane tiltrotor has been presented in Ref 13, using the analytical models developed and validated in this paper.

Summary and Conclusions

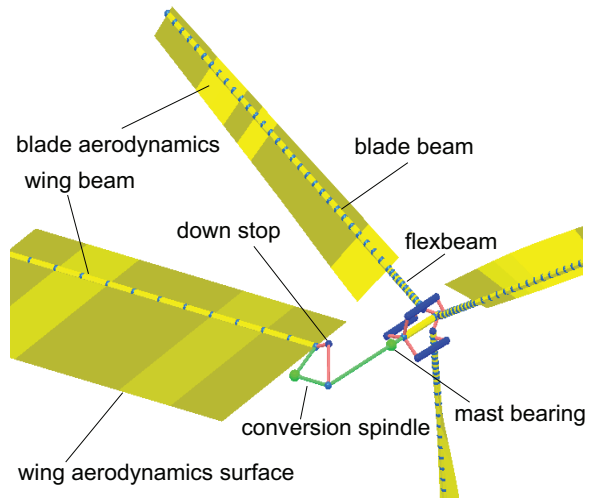
The paper presents the development and validation of a stiff-inplane tiltrotor model using two multibody rotorcraft codes. The model development and validation methodologies by modularization are emphasized in the framework of multibody dynamics simulation. Validation procedure and results for key sub-components are presented. The multibody dynamics models developed in this effort include the gimbaled hub, rotor blades, pitch links, swashplate, conversion actuators which are attached to the pylon, and the elastic wing. The natural frequencies, mode shapes, and stability characteristics of key sub-component structures and the kinematic couplings of rotor system are correlated with the predictions of the two analyses. Furthermore, the experimental data obtained in the ground vibration test and wind-tunnel test is extensively used to validate the analytical models. The analytical models are then used to predict the whirl-flutter stability boundary, which show good agreement with the experimental measurements. The predicted flutter speeds by both analyses are within 8% of the measured value for rotor speed of 742 RPM (airplane mode rotor speed.) This study showed that modularization in model developing and validation, that is the development of sophisticated rotorcraft models component by component as well as the validation of the model component by component, is a reliable method to build the analytical model accurately and efficiently.

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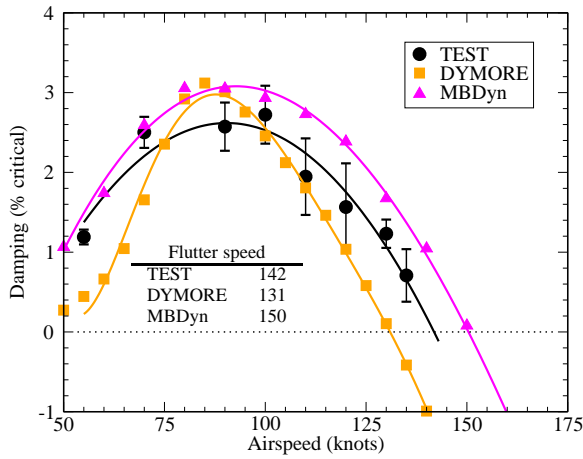
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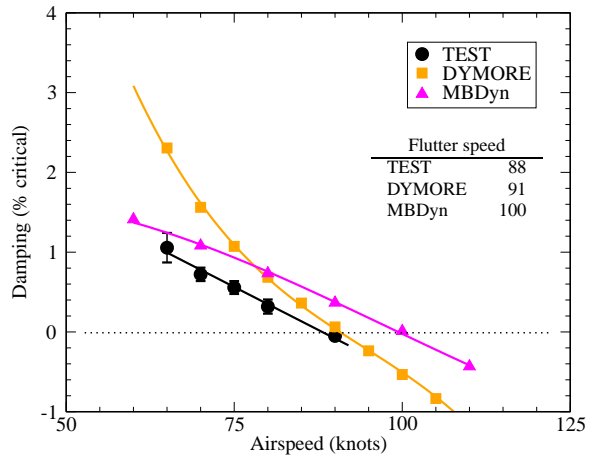
(a) WRATS stiff-inplane tiltrotor in TDT tunnel.



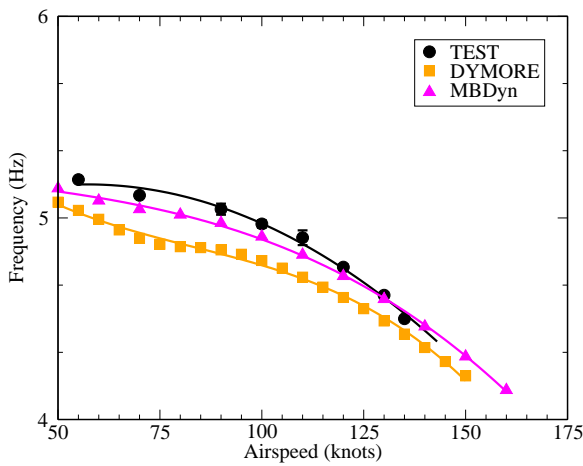
(b) WRATS stiff-inplane tiltrotor MDS model.



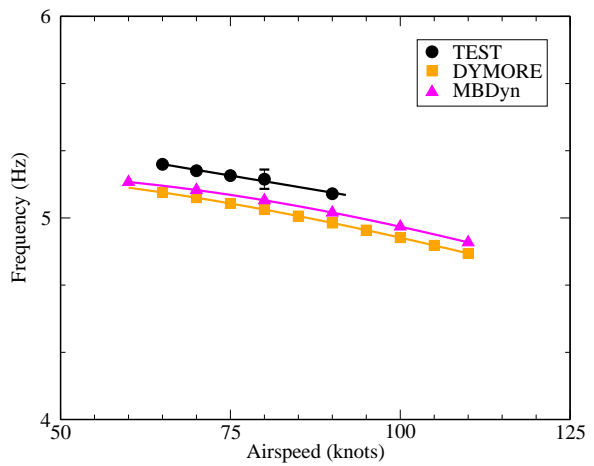
(c) Wing beam mode damping (742 RPM.)



(d) Wing beam mode damping (888 RPM.)



(e) Wing beam mode frequency (742 RPM.)



(f) Wing beam mode frequency (888 RPM.)

Figure 10: Validation of the complete WRATS semi-span stiff-inplane tiltrotor model.