DETERMINING ATMOSPHERIC DENSITIES AND SPACECRAFT AERODYNAMIC PROPERTIES FROM MAGELLAN ATTITUDE DATA

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Abstract

A method is examined for calculating density of the Venusian atmosphere and improving spacecraft aerodynamic properties by using spacecraft reaction wheel data. The orbit brings Magellan into the upper reaches of the atmosphere where daytime densities of 0.5 x 10^{-13} g/cm^3, can induce torques on the order of 0.02 N-m. The attitude control system keeps the spacecraft inertially fixed by changing the angular rates of three reaction wheels. By modeling the Magellan trajectory and applying a least squares solution technique to the reaction wheel data, atmospheric density and the two unknown directional moment coefficients can be determined. The densities derived from this technique correspond closely with those developed from doppler tracking data, however, the moment coefficients do not agree with the free molecular flow calculations.

Nomenclature

A_{ref}	reference area (23 m^2)
$B_{y,p,r}$	reaction wheel bias
C_m	moment coefficient
Γ	measurement covariance matrix
h	altitude, km
h_0	base altitude above Venus, km
H_s	scale height, km
I	spacecraft inertia matrix, kg-m^2
L_{ref}	reference length (3.66 m)
M_i	i-th model data point
MNT	Magellan Navigation Team
μ_{venus}	gravitational constant, km^3/s^2
ρ	atmospheric density, g/cm^3
ρ_0	base density, g/cm^3
σ	measurement standard deviation
T_{drag}	drag torque, N-m
T_{grav}	gravity gradient torque, N-m
t_i	time corresponding to i-th data point
v	spacecraft velocity, m/s
Y_i	i-th observed data point

Introduction

The Magellan spacecraft was launched from the space shuttle *Atlantis*, STS-30, on May, 4 1989, and was injected into a near polar orbit around Venus on August 10, 1990. The primary mission was to map the surface of the planet. Due to the opaque nature of the Venusian atmosphere, imaging by synthetic aperture radar was used. During the first three Venusian sidereal days, ending on September 14, 1992, over 97% of the planet's surface had been mapped. The fourth Venusian sidereal day (Sept. 15, 1992 to May 16, 1993) is reserved for mapping the gravitational field and upper atmospheric structure. Both the gravitational field and atmospheric structure are being determined from doppler tracking, using traditional methods.

The purpose of this paper is twofold: (1) to present a feasibility study of an alternate method of computing the atmospheric density based on data from the spacecraft attitude control system and compare it to densities derived by the Magellan Navigation Team at the Jet Propulsion Lab using the doppler data, and (2) to present a method for experimentally determining the aerodynamic moment coefficients for two principal spacecraft axes. Accurate knowledge of the Venusian atmosphere and aerodynamic characteristics of the spacecraft is important since Magellan will begin a limited aerobraking maneuver to circularize the orbit in late May, 1993.[1] Although not designed for aerobraking, it is the only technique, given the limited fuel reserves, which will lower the apoapsis

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and allow greater resolution in the gravity field measurements.[2]

The spacecraft, shown in figure 1, has a mass of 1080 kg. After an orbital maneuver on Sept. 15, 1992, the orbit has a eccentricity of 0.3995 and an inclination with respect to the mean Venus equator of 85.5°. The semimajor axis is 10387 km. With the Venus reference radius as 6051 km, the apoapsis altitude is 8487 km and periapsis altitude is 185 km. The 185 km periapsis allows for atmospheric and gravity studies unobtainable at higher altitudes. Prior to the maneuver, the periapsis was near 300 km for radar mapping purposes. When aerobraking is initiated, the periapsis will be lowered to near 140 km to allow drag to rapidly remove energy from the orbit.[2] For periapsis altitudes near 185 km, drag is apparent and reduces the semimajor axis by approximately 8.0 m per orbit.

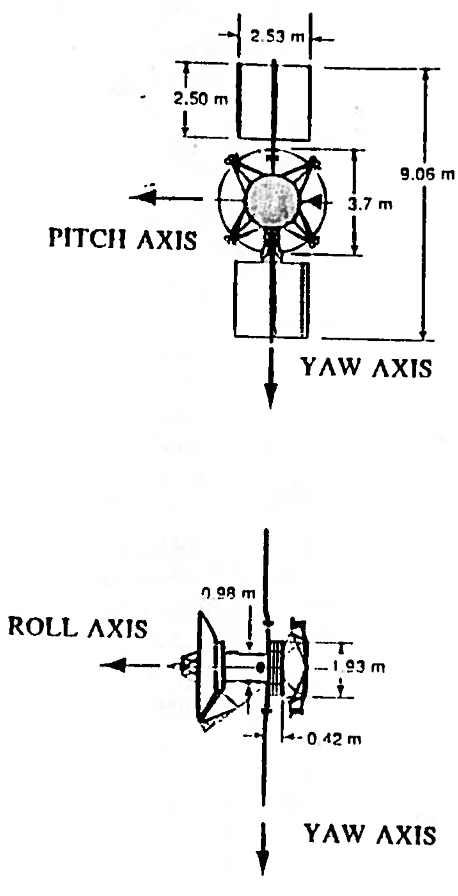

Figure 1. Magellan Spacecraft

The Atmosphere

The Venusian upper atmosphere was previously measured by Pioneer Venus.[3] *In situ* measurements from Pioneer Venus and remote sensing from Veneras 11 and 12 show the atmosphere to be composed of H, He, N_2, N, CO, CO_2, and O, the most numerous species being carbon dioxide. The structure and composition is given in the *Venus International Reference Atmosphere, 1985 (VIRA)*.[4] Above 150 km, each species is assumed to be in diffusive equilibrium. The VIRA model is a highly parameterized model and therefore, not appropriate for least square methods. Instead, a simplified discrete model is used. This model divides the atmosphere into five kilometer layers. Within each of these layers, the scale height is considered constant. The simplified model of the Venus atmosphere is expressed by,

$$\rho=\rho_0 e^{-\left(\frac{h-h_0}{H_i}\right)} \quad (1)$$

Knowing the scale height structure and a base density allows the density at any altitude to be calculated. For both the technique using doppler data and the technique presented in this paper, the scale height structure from the VIRA is assumed. Figure 2 shows density as a function of altitude for noon and 4:00 PM local solar time. Figure 3 shows the scale height for the same time periods. Both the figures use values derived for a F10.7 cm solar flux of 150. The symmetric VIRA model used for both the doppler and reaction wheel methods assumes the densities are symmetric in time about local solar noon. This implies that the scale heights are also symmetric about noon. Using this structure, any density at periapsis can be used to determine a base density. The base density can then be used to construct the entire density profile for the region.

Data Transmission, Reaction Wheel, and Control System Specifics

Reaction wheel tachometer data is read at 0.9375 Hz and can be transmitted at four possible sample rates. The rates are 0.666 seconds, 8.666 seconds, 20 seconds, and 240 seconds.[5] Magellan attitude quaternions are transmitted every 0.666 seconds and body rates every 8.666 seconds. The archive database protocol dictates telemetry quantities are stored only if they change from one

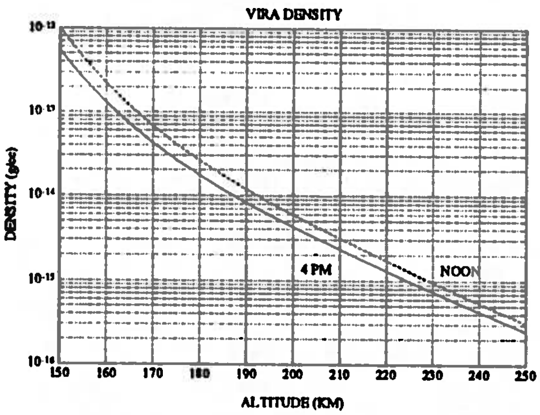

Figure 2. Noon and 4:00 PM VIRA Densities

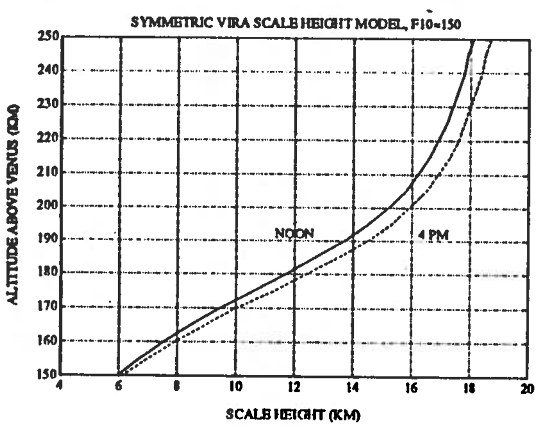

Figure 3. Noon and 4:00 PM VIRA Scale Heights

frame to the next. Therefore, data which has been omitted must be reconstructed prior to analysis.

The three reaction wheels are used for fine attitude adjustment. The reaction wheels are normally desaturated every orbit approximately 15 minutes after periapsis using the RCS jets. The wheel characteristics are listed in table 1.

Inertia = 0.06638 N-m-s^2
Max Torque Output = 0.18 N-m
Max Momentum Change = 27.0 N-m-s
Tachometer Quantization = 1.02 rad/s

Table 1. Reaction Wheel Characteristics

Examination of the error between commanded quaternions and actual quaternions showed no observable pointing errors during the periapsis passage. Therefore, the response of the spacecraft was assumed to have zero time lag and zero overshoot compared to the commands of the control system. Hence, it is not necessary to model the dynamic response of spacecraft attitude.

The tachometers use a Hall effect sensor and three commutators set on the wheel. The meter senses the magnetic field variations from the commutators and deduces an angular velocity for the wheel. This signal is updated every 1.07 seconds. Due to dynamic friction, the tachometer system has some difficulty obtaining accurate measurements when the wheel angular velocity is near zero.[6] Accordingly, residuals that result from data taken at low wheel speeds tend to be slightly greater than those taken at higher speeds.

Environmental Torques

The torques considered for the simulation were gravity gradient, solar pressure, and atmospheric drag. The gravity gradient torques were calculated using Eq. 2,

$$T_{grav}=\frac{3\mu_{venus}}{R^3}\{[mn(I_{zz}-I_{yy})+lnI_{xy}-lmI_{xz}+(n^2-m^2)I_{yz}]\,\vec{i}$$
$$+[nl(I_{xx}-I_{zz})+lmI_{yz}-mnI_{xy}+(l^2-n^2)I_{xz}]\,\vec{j}$$
$$+[lm(I_{yy}-I_{xx})+mnI_{xz}-lnI_{yz}+(m^2-l^2)I_{xy}]\,\vec{k}\,\} \quad (2)$$

Where R is the distance from the planet center of mass to the spacecraft center of mass and l,m,n are the direction cosines of $\vec{R}$ with respect to the spacecraft coordinate frame.[7] Table 2 shows the moments of inertia of the spacecraft.

I_{xx} = 1101.7 kg m^2
I_{yy} = 2031.3 kg m^2
I_{zz} = 1543.4 kg m^2
I_{xy} = 2.55 kg m^2
I_{xz} = 37.4 kg m^2
I_{yz} = 1.10 kg m^2

Table 2. Spacecraft Moments of Inertia

Maximum torques due to gravity were computed as being on the order of 0.01 N-m or a cumulative reaction wheel rate change of 5 rad/s over one periapsis pass.

Drag torques were computed using Eq. 3.[8]

$$T_{drag}=\frac{1}{2}\rho v^2 C_m A_{ref} L_{ref} \quad (3)$$

Due to the orientation at periapsis, the primary aerodynamic torque effect occurs in the pitch axis. Initially, the moment coefficient for that direction was taken from numerical modeling done by Schmitt (figure 4)[9]. During the periapsis pass the angle of attack, which is measured from the negative roll axis, varies between 255° and 285°.

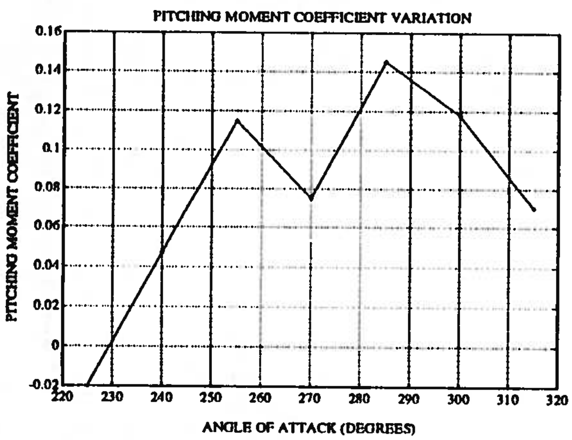

Figure 4. Pitch Moment Coefficients (After Schmitt, MPT Presentation 11-1-91)

Solar pressure torque was found to be less than 2% of either gravitational or aerodynamic torques. It was, therefore, neglected in the analysis.

Data Analysis

The simulation of the trajectory was done in Fortran on an IBM 386 PC. The orbits presented in this paper were from November 7 to November 23, 1992. The local solar time at periapsis varied from 13:17 to 15:40. Because of the small decay of the semimajor axis, the orbit was assumed Keplerian over the course of the periapsis pass and simulation times generally ran for 10 minutes on either side of closest approach. The spacecraft state vector was propagated in 15 second intervals. The orientation was determined using quaternions from the telemetry and was found to remain constant over the simulated time. All torques were computed at each time step and summed.

At each point in the simulation the atmospheric density was calculated assuming the above described scale height models taken from the discrete representation of the *Venus International Reference Atmosphere* and an initial guess of the base density.

The estimated base density was calculated using the least squares method described below. A linear interpolation of the pitch axis model data was done such that both the observed data and model had values at the same points in time. With this, the residuals between the two sets could be found. By minimizing these residuals, the bias and base density was calculated. Biases are initial wheel speeds in the observed data which arise before the start of the simulation. These rates are most often due to wheel desaturation or spacecraft maneuvering during the unsimulated portion of the orbit. With the new base density the simulation was run again and a similar procedure was followed with the roll and yaw axes data such that the biases and moment coefficients were found. A sample simulation for orbit #6211 is illustrated by figure 5 which shows the actual and modeled reaction wheel speeds plotted versus time. The rapid rise in the pitch axis wheel is due to the atmospheric torque. The variations before and after the 400 second period around periapsis are the result of gravity gradient torques.

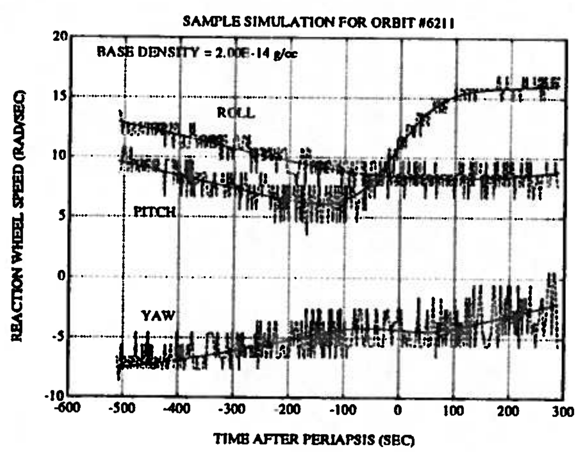

Figure 5 Reaction Wheel Rates

Least Squares Solution

After the time matching of the model and observed data sets, the residuals were found. Residuals being defined as,

$$R_i = Y_i - M_i \tag{4}$$

Noting that the pitch axis reaction wheel rates are a function of base density and initial bias, partial derivatives for each data point can be found.

$$A_p = \begin{bmatrix} \frac{\partial M_p}{\partial B_p}|t_1 & \frac{\partial M_p}{\partial \rho_0}|t_1 \\ \frac{\partial M_p}{\partial B_p}|t_2 & \frac{\partial M_p}{\partial \rho_0}|t_2 \\ \vdots & \vdots \\ \frac{\partial M_p}{\partial B_p}|t_n & \frac{\partial M_p}{\partial \rho_0}|t_n \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2}C_m v^2 A_{ref} L_{ref}|t_1 \\ 1 & \frac{1}{2}C_m v^2 A_{ref} L_{ref}|t_2 \\ \vdots & \vdots \\ 1 & \frac{1}{2}C_m v^2 A_{ref} L_{ref}|t_n \end{bmatrix} \tag{5}$$

Here, n is the number of data points and M_p are the model pitch axis data points, such that,

$$\{R_p\} = [A_p] \begin{Bmatrix} \delta B_p \\ \delta \rho_0 \end{Bmatrix} \tag{6}$$

where δB_p and $\delta \rho_0$ are the differences between model and observed bias and model and observed base density, respectively. Using a minimum variance estimator and solving for the δ's,

$$\begin{Bmatrix} \delta B_p \\ \delta \rho_0 \end{Bmatrix} = \left(A_p^T \Gamma^{-1} A_p\right)^{-1} A_p^T \Gamma^{-1} \{R_p\} \tag{7}$$

where the Γ is the covariance matrix for the observed data.[10] Γ for all data rates other than 0.666 seconds is a diagonal matrix, because the sampling rate from the tachometers is higher than the transmission rate. Subsequently, there is no correlation between successive data points and all off diagonal elements of Γ are zero. Γ for the 0.666 second data rate is given by Eq. 8. This tridiagonal matrix comes from an autocorrelation study of the residuals. Adjusting the model base density by $\delta \rho_0$ and the bias by δB_p and rerunning the simulation will drive the pitch axis residuals to a minimum.

$$\Gamma = \begin{Bmatrix} 1 & .3 & 0 & \cdots & \cdots & \cdots & 0 \\ .3 & 1 & .3 & & & & \vdots \\ 0 & .3 & 1 & \ddots & & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & 1 & .3 & 0 \\ \vdots & & & & .3 & 1 & .3 \\ 0 & \cdots & \cdots & \cdots & 0 & .3 & 1 \end{Bmatrix} \sigma^2 \tag{8}$$

Once the base density has been found, simulations for the roll and yaw axis are performed. The same procedure is followed except that the least squares solution is found for the moment coefficients and the biases in those directions. With the fixed base density, the A matrices for roll and yaw become,

$$A_r = \begin{bmatrix} \frac{\partial M_r}{\partial B_r}|t_1 & \frac{\partial M_r}{\partial Cm_r}|t_1 \\ \vdots & \vdots \\ \frac{\partial M_r}{\partial B_r}|t_n & \frac{\partial M_r}{\partial Cm_r}|t_n \end{bmatrix}$$

$$A_y = \begin{bmatrix} \frac{\partial M_y}{\partial B_y}|t_1 & \frac{\partial M_y}{\partial Cm_y}|t_1 \\ \vdots & \vdots \\ \frac{\partial M_y}{\partial B_y}|t_n & \frac{\partial M_y}{\partial Cm_y}|t_n \end{bmatrix} \tag{9}$$

such that,

$$\{R_r\} = [A_r] \begin{Bmatrix} \delta B_r \\ \delta Cm_r \end{Bmatrix}, \quad \{R_y\} = [A_y] \begin{Bmatrix} \delta B_y \\ \delta Cm_y \end{Bmatrix} \tag{10}$$

Using the same method as the base density, the roll and yaw biases and the moment coefficients were determined so that the weighted residuals in those directions are also minimized.

Doppler Shift Method

The densities to which the reaction wheel based numbers will be compared are those derived from analyzing the doppler data. The method used by MNT also involves a least squares fit to the range rate data. Eq. 11 is analogous to Eq. 6 above, or symbolically,

$$\{R_{dop}\} = \left[\frac{\partial \dot{r}}{\partial \rho_0}\right]\{\delta\rho_0\} \tag{11}$$

where R_{dop} is the residuals between the expected and measured doppler shifts, and $\dot{r}$ is the range rate data. In order to evaluate the partial derivative, a C_D and a scale height structure must be assumed. The C_D was chosen as a constant 2.2 regardless of orientation and the symmetric VIRA scale height model, in the same form as above, was used.

Results

The MNT densities and the densities derived from reaction wheel data correspond closely over the orbit interval examined. Figure 6 shows the density values while figure 7 illustrates the ratio of the densities. The MNT derived densities are between 10% and 30% higher than reaction wheel derived numbers for orbits up to #6250.

While the general shape of the two base density curves match, the constant offset indicates that the pitch axis coefficient is overestimated by figure 4. Martin Marietta Astronautics Group subsequently ran several Magellan orientations using the FREEMAC free molecular flow code. The results of these simulations are shown in table 3.[11] This program included the orientation of the solar panels and spacecraft roll not considered previously. With the improved pitch axis aerodynamics, the reaction wheel simulation was run again. This time, as illustrated by figures 8 and 9, the base density curves were generally within 10% of the MNT numbers for orbits #6182 through #6242. The orbits after #6242 show a difference of near 20% with the reaction wheel densities lower. The orbits prior to #6182 show a 10% to 25% variation from the MNT derived numbers.

Figures 10 and 11 illustrate the yaw and roll moment coefficients as recovered from the least squares solution for the reaction wheel data. The yaw coefficients are approximately 50% larger than the FREEMAC results for orbits prior to #6242 and show a substantial discrepancy for orbits after #6242. The roll coefficients are an order of ten larger than those predicted by FREEMAC for orbits before #6182. For the remainder of the orbits, the least squares derived coefficients are approximately one third to one fourth the predicted values.

ORBIT	CD	CM(p)	CM(r)	CM(y)
6148	0.680	0.071	-0.001	-0.010
6189	0.676	0.072	0.007	-0.009
6211	0.680	0.073	0.007	-0.009
6462	0.725	0.092	0.003	0.016

Table 3. FREEMAC Results

Conclusions

The densities from the method using reaction wheel data corresponds closely to the MNT numbers for the orbits #6182 through #6242. For the regions before #6182 and after #6242 the densities are not in as good agreement. Both of those regions correspond to changes in the cross sectional area of the spacecraft, either by orientation change or solar panel rotation. The nominal orientation of Magellan is with the yaw axis perpendicular to the Sun-Earth-Venus plane, the roll axis pointed toward Earth, and the solar arrays in sun-tracking mode. This is the orientation during the #6182-#6242 period. Prior to #6182, the solar panels were rotated by 45° to the sun for cooling purposes. After #6242, the spacecraft had been rolled +10°. Both of these area changes were not considered in the MNT analysis. The reaction wheel method does take in account these area changes by including the quaternions, *i.e.* attitude, in the determination of the pitch axis moment coefficient and, therefore, the base density. This is a possible explanation for the difference between the MNT and reaction wheel derived densities after orbit #6242.

The method of using reaction wheel data may represent as accurate a way of extracting densities of an upper atmosphere as the doppler technique. It may also serve as a check for free molecular flow simulation programs.

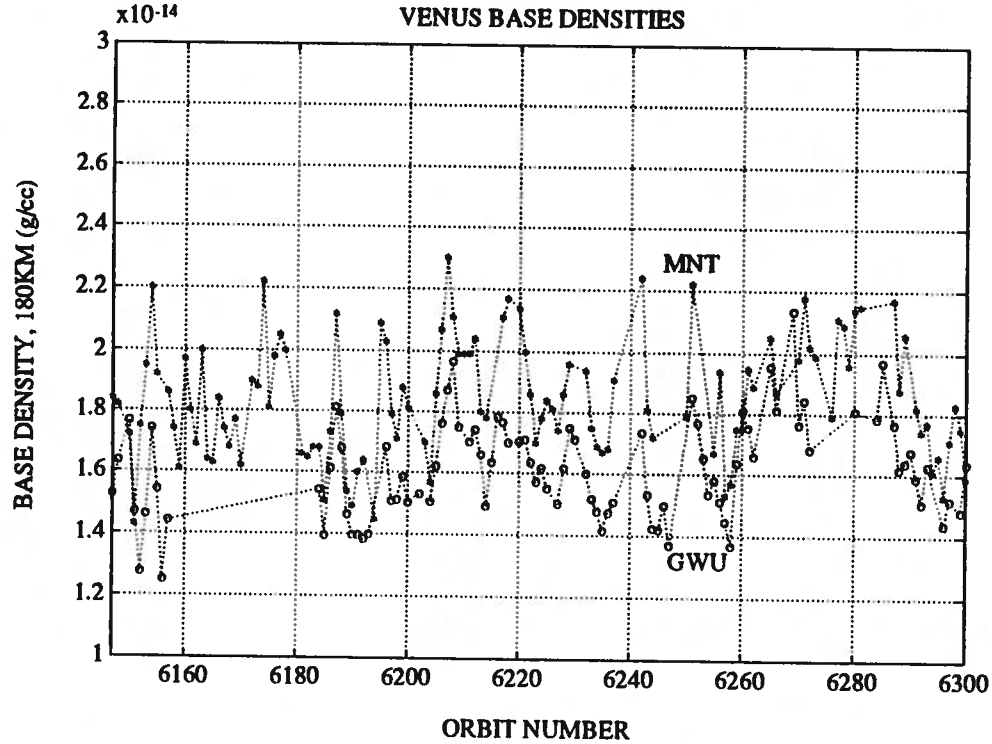

Figure 6. Base Densities

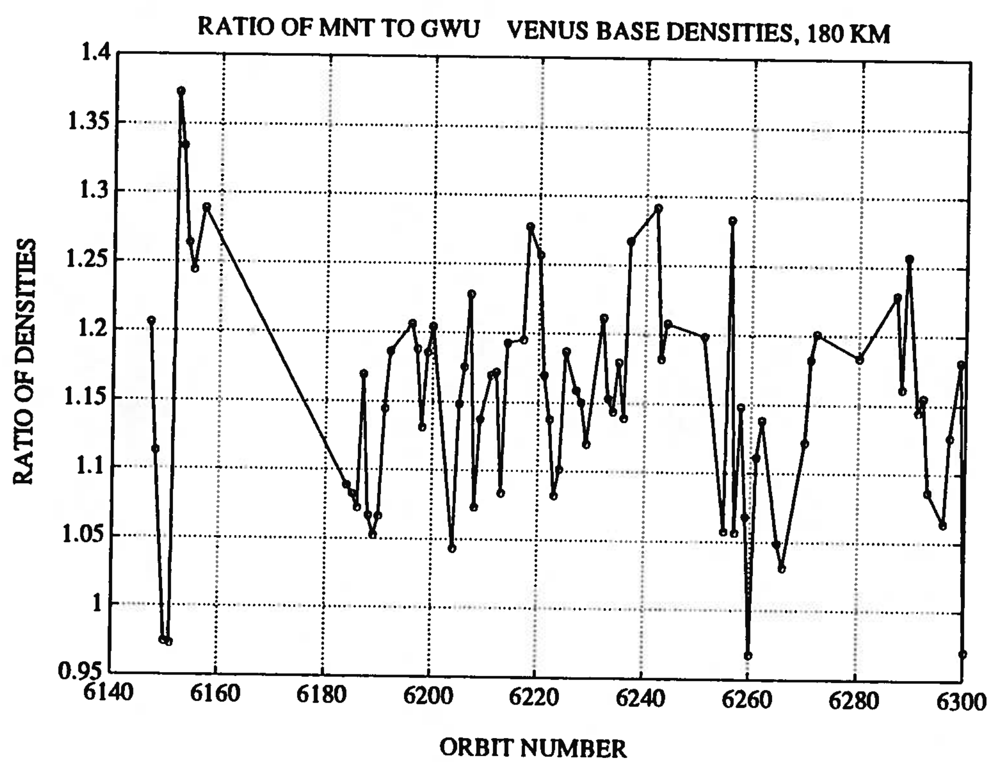

Figure 7. Ratio of Base Densities MNT / GWU

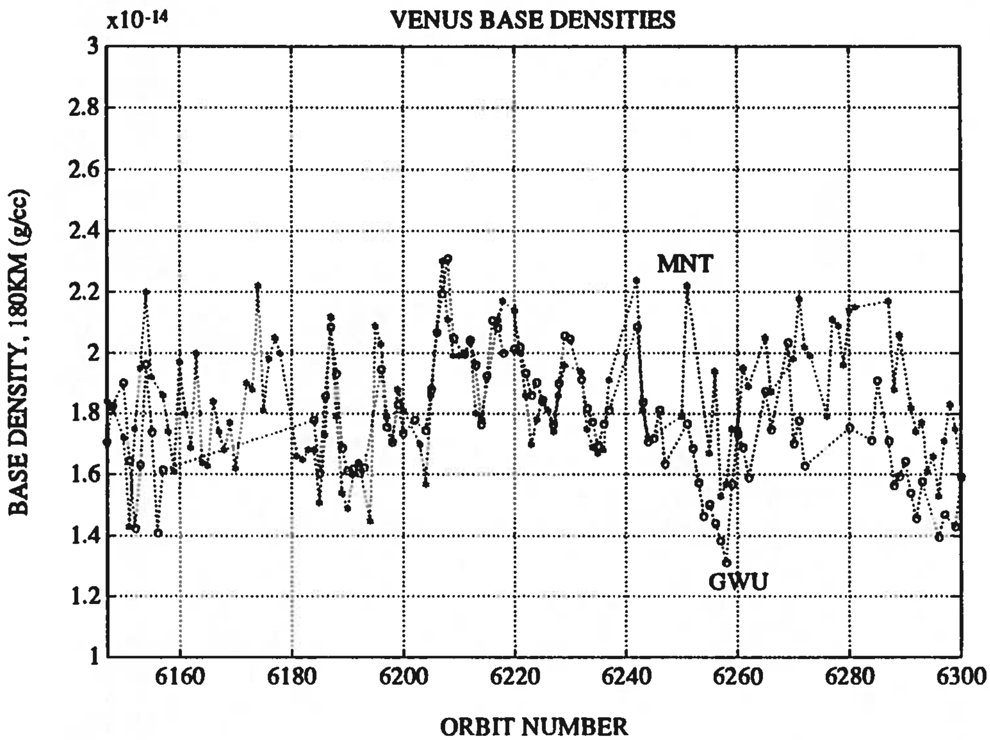

Figure 8. Base Densities

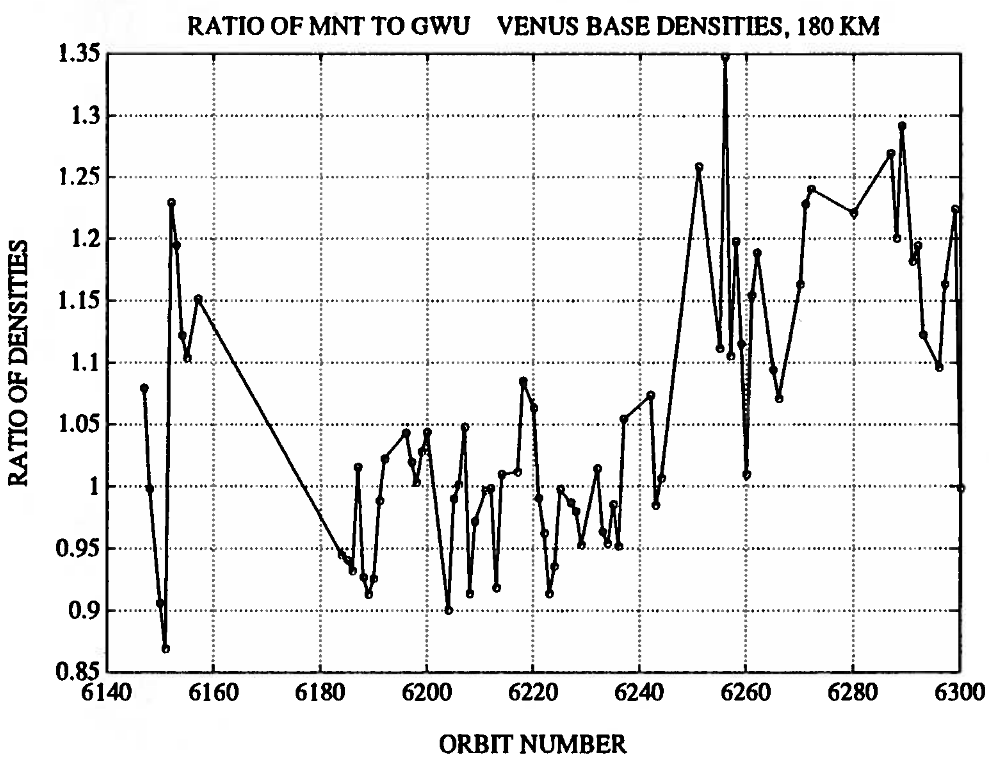

Figure 9. Ratio of Base Densities MNT / GWU

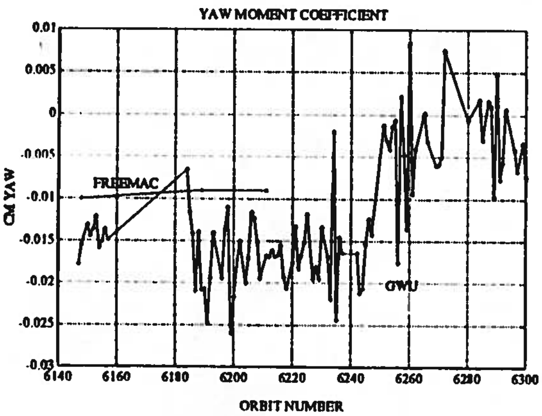

Figure 10 Yaw Moment Coefficients

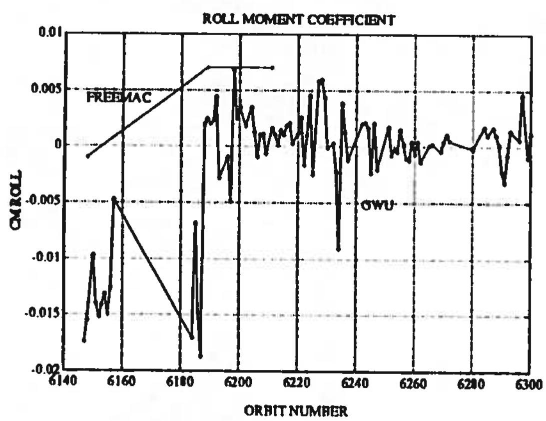

Figure 11 Roll Moment Coefficients

Future Work

Unlike the doppler method, the reaction wheel method may hold enough information to help develop more representative scale height models. At present, the wheel data from a single orbit does not have the resolution necessary to identify scale height effects. A possible way of extracting scale height information is to assume the atmosphere variability over 24 hours is negligible. This would allow the combining of reaction wheel data from several orbits in an attempt to ascertain effects caused by errors in the scale height model. Also, the exact effect of the uncertainties in the aerodynamics should be mapped into density uncertainties.

Acknowledgements

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