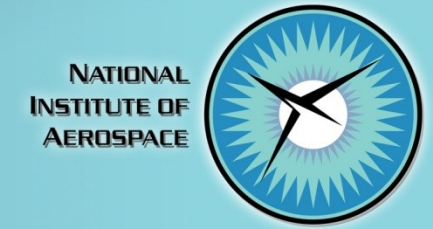


New-Generation CFD Codes by *Hyperbolic* Navier-Stokes System



Objective: Create *new-generation* Computational Fluid Dynamics (CFD) codes that are orders-of-magnitude more efficient and accurate than existing (traditional) codes by revolutionizing the way we solve partial-differential equations numerically. “Turn it into hyperbolic.”

Approach: **First-Order Hyperbolic System Method** Nishikawa, JCP2007
Nishikawa, JCP2010

Diffusion Equation (**Parabolic**)

First-Order System (**Hyperbolic**)

$$u_t = \nu u_{xx}$$

$$u_t = \nu p_x$$

$$p_t = (u_x - p)/Tr$$

Two models are equivalent in the steady state for arbitrary relaxation time, Tr .

We compute the steady state solution to the diffusion equation by integrating the hyperbolic system in time to the steady state.

Intrinsic Features:

1. Methods for hyperbolic systems are directly applicable to diffusion.

If you have a good method/textbook for hyperbolic systems, you have a good method/textbook for the diffusion equation. Hard to believe, but now true.

2. $O(1/h)$ speed-up to obtain the steady state solution.

Time step is not $O(h^2)$ but $O(h)$ because the system is first-order (h is a mesh spacing). The speed-up factor is $O(1/h)$: it gets **faster** for larger-scale problems.

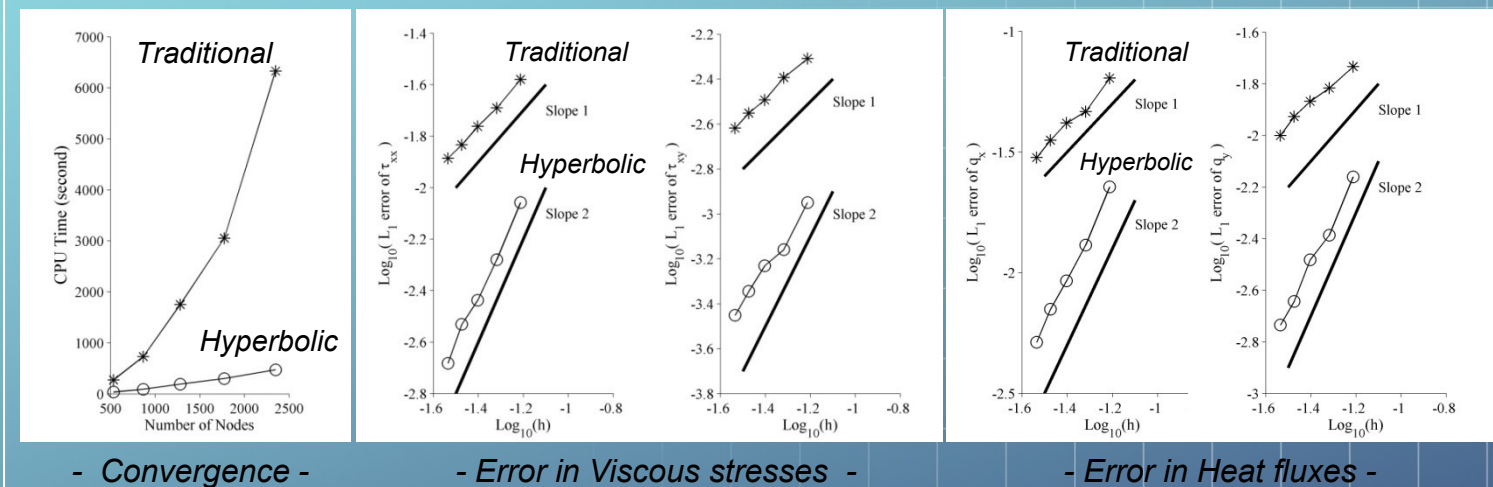
3. Higher-order accuracy in the solution gradient, p .

The solution gradient is now an additional unknown. It'll be computed to the same order of accuracy as that of the solution (one order less otherwise).

Results:

Nishikawa, AIAA Paper 2011-3043

The first-order hyperbolic system method has been extended to the Navier-Stokes system by writing the viscous term as a first-order system with viscous/heat fluxes added as additional unknowns. Left figure shows the CPU time comparison obtained for 2D irregular grid calculations by the hyperbolic method and a traditional method. The hyperbolic method is faster, and gets faster for finer meshes. Right figure shows the error convergence on the viscous/heat fluxes. The hyperbolic method gives 2nd-order accurate viscous/heat fluxes while the traditional method gives only 1st-order accuracy. The results show *orders-of-magnitude* improvements by the hyperbolic method.



- Convergence -

- Error in Viscous stresses -

- Error in Heat fluxes -

We've just opened the door to the next-generation CFD codes. It is only the beginning.

Research Interests:
Algorithm development for
Computational Fluid Dynamics.

Hiro(aki) Nishikawa, Ph.D.
Senior Research Scientist
hiro@nianet.org

